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R^2 corrections to asymptotically Lifshitz spacetimes

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Abstract: We study R^2 corrections to five-dimensional asymptotically Lifshitz spacetimes by adding Gauss-Bonnet terms in the effective action. For the zero-temperature backgrounds we obtain exact solutions in both pure Gauss-Bonnet gravity and Gauss-Bonnet gravity with non-trivial matter. The dynamical exponent undergoes finite renormalization in the latter case. For the finite-temperature backgrounds we obtain black brane solutions perturbatively and calculate the ratio of shear viscosity to entropy density η/s. The KSS bound is still violated but unlike the relativistic counterparts, the causality of the boundary field theory cannot be taken as a constraint.

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Contents

1 Introduction

The AdS/CFT correspondence [1–3] relates conformal field theories to gravitational dynamics in asymptotically AdS backgrounds. As a strong-weak duality, it has yielded many important insights into the dynamics of strongly coupled field theories. For instance, the hydrodynamic behavior of finite-temperature field theory can be reflected in the dual gravity side [4]. Recently, there has been a great deal of progress in applying the AdS/CFT correspondence to study condensed matter systems near a critical point, for reviews see [5].

There are many scale-invariant field theories in which time and space can scale differently,

$$t \;\to\; \lambda^z t, \qquad x^i \;\to\; \lambda x^i, \tag{1.1}$$

where z is called the 'dynamical exponent'. Such field theories with anisotropic scaling symmetry are of interest in studying condensed matter systems near a critical point. The corresponding gravity duals have been investigated extensively and there are mainly two concrete cases of interest till now. One is the theory with Galilean boost symmetry as well as anisotropic scaling symmetry, whose symmetry group is the Schrödinger group for $z = 2$. The gravity duals were obtained in [6, 7] with the following metric

$$ds^2 = -r^4(dx^+)^2 - 2r^2dx^+dx^- + \frac{dr^2}{r^2} + r^2d\vec{x}^2. \tag{1.2}$$

The embedding of this spacetime into string theory and the finite-temperature generalizations have been successfully realized in [8–11]. Such backgrounds, both the zero-temperature case and the finite-temperature case, can be obtained by performing the "Null Melvin Twist" [12] on the corresponding (black) D-brane configurations.

The other one has no boost symmetry, which is known as the Lifshitz case, and the gravity duals were obtained in [13]

$$ds^2 = L^2\left(-r^{2z}dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}^2\right). \tag{1.3}$$

Some other gravity solutions with similar anisotropic scale invariance were studied in [14]. Unlike the Schrödinger case, it is quite difficult to embed the Lifshitz background into string theory, so is to find the finite temperature generalizations. The embedding of Lifshitz-like fixed points into type IIB string theory was discussed extensively in [15], based on the D3-D7 solutions introduced in [16]. Since the dilaton in the solution is not constant, the anisotropic scale invariance only holds at the leading order of interactions. Some no-go theorems for string duals of non-relativistic Lifshitz-like theories were proposed quite recently in [17], where the authors argued that such gravity duals in the supergravities were not possible. Black hole in asymptotically Lifshitz spacetimes were discussed in [18–21], where most of the solutions were obtained numerically and exact solutions could be found only in certain specific examples.

Several aspects of non-relativistic holography were studied in [22], where it was observed that the Lifshitz geometry was a solution of a gravity theory coupled with a massive vector. Furthermore, it was found that the following action

$$S = \frac{1}{16\pi G_{d+2}}\int d^{d+2}x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{\lambda\phi}F_{\mu\nu}F^{\mu\nu}\right] \tag{1.4}$$

admitted a solution with anisotropic scaling symmetry

$$\begin{aligned}
&ds^2 = L^2\left(-r^{2z}dt^2 + \frac{dr^2}{r^2} + r^2\sum_{i=1}^{d} dx_i^2\right), \\
&F_{rt} = qe^{-\lambda\phi}r^{z-d-1}, && e^{\lambda\phi} = r^{\lambda\sqrt{2(z-1)d}}, \\
&\lambda^2 = \frac{2d}{z-1}, && q^2 = 2L^2(z-1)(z+d), \\
&\Lambda = -\frac{(z+d-1)(z+d)}{2L^2}.
\end{aligned} \tag{1.5}$$

The metric is Lifshitz-like but the dilaton is not constant, so it cannot be seen as a genuine gravity dual of Lifshitz-fixed points. However, such a solution is worth investigating, as it also possesses exact solutions with finite temperature. Some properties of the corresponding black branes were discussed in [23].

In this paper we will study R^2 corrections to the above mentioned Lifshitz-like gravity backgrounds. The $1/N$ effects in non-relativistic gauge-gravity duality was investigated extensively in [24], where they argued that the dynamical exponent received finite renormalization and the ratio of shear viscosity to entropy density weakly violated the celebrated

KSS bound $1/4\pi$. Here we first study the R^2 corrections to the zero-temperature cases by solving the equations of motion explicitly. We find that an exact solution of Lifshitz background can be found in pure Gauss-Bonnet gravity, while the dynamical exponent undergoes a finite renormalization when non-trivial matter fields are included. We also obtain the corresponding black brane solutions by perturbative methods and calculate the ratio of shear viscosity to entropy density. The result also violates the KSS bound and it reduces to the known result when the dynamical exponent $z = 1$. However, since the boundary field theory is non-relativistic, the causality of the field theory cannot be taken as a constraint on the ratio.

The rest of the paper is organized as follows: In section 2 we review some necessary backgrounds, including the renormalization of the dynamical exponent and the solutions in asymptotically Lifshitz spacetimes. The R^2 corrections to the zero-temperature case are studied in section 3 and the corrections to the black brane case are studied in section 4. We calculate the ratio of shear viscosity to entropy density in section 5 through the effective coupling of the transverse gravitons in the dual gravity side. A summary and discussion will be given in the final section.

2 Some backgrounds

In this section we will review some backgrounds which are necessary for further investigations. One of the main results in [24], that is, both the radius of curvature and the dynamical exponent z may be renormalized for non-relativistic Lifshitz metric, will be summarized in section 2.1. The zero-temperature and black brane solutions in asymptotically Lifshitz spacetimes, obtained in [22], will be reviewed in section 2.2.

2.1 Solutions with Lifshitz symmetry and invariant two-forms

The renormalization of the curvature radius and the dynamical exponent for both Schrödinger and Lifshitz metrics was demonstrated in [24] in a beautiful way. Here we just focus on the Lifshitz case, whose symmetry algebra contains the following generators: Hamiltonian H, linear momenta P_i, angular momenta M_{ij} and a dilaton operator D. The commutators among H, P_i and M_{ij} behave the same as usual, and the dilaton operator D has the following non-trivial commutators

$$[D, P_i] = iP_i, \quad [D, H] = izH. \tag{2.1}$$

We rewrite the $(d + 2)-$dimensional gravity duals of Lifshitz fixed points in [13] as

$$ds^2 = L^2\left(-\frac{dt^2}{r^{2z}} + \frac{dr^2 + d\vec{x}^2}{r^2}\right). \tag{2.2}$$

The corresponding Killing vectors are

$$H = -i\partial_t, \quad P_i = -i\partial_i, \quad M_{ij} = -i(x^i\partial_j - x^j\partial_i), \quad D = -i(zt\partial_t + x^i\partial_i + r\partial_r). \tag{2.3}$$

Since we are trying to find Lifshitz-like solutions to the equations of motion of gravity coupled to some matter sector, the Einstein tensor $G_{\mu\nu}$ and the stress tensor $T_{\mu\nu}$ are

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symmetric two-tensors invariant under the Lifshitz symmetries (2.3). The search for solutions may be simplified if we expand the Einstein equations in a basis of such symmetric invariant two-forms. Let $\tau = \tau_{\mu\nu}dx^\mu dx^\nu$ be a symmetric two-tensor invariant under the Lifshitz symmetries, i.e. $\mathcal{L}_v \tau_{\mu\nu} = 0$ for all the Killing vectors v in (2.3). The symmetry of the two-tensor plus the conservation of the stress tensor $\nabla^\mu \tau_{\mu\nu} = 0$ imply

$$\tau = \alpha \frac{dt^2}{r^{2z}} + \beta \frac{d\vec{x}^2}{r^2} + \left(\frac{(d-2)\beta - z\alpha}{d-2+z} \right) \frac{dr^2}{r^2}, \tag{2.4}$$

where α, β are two constants. Thus it is a two-parameter family of conserved stress tensors.

Consider the action of gravity coupled to an arbitrary matter sector

$$S = \frac{1}{16\pi G} \int \sqrt{-g}(R - 2\Lambda) + S_m(g_{\mu\nu}, \phi_i), \tag{2.5}$$

where S_m denotes the matter part of the action and ϕ_i stand for the matter fields. S_m can also contain higher derivative corrections to the Einstein-Hilbert action. The equations of motion are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}, \tag{2.6}$$

where $T_{\mu\nu} = \delta S_m / \delta g^{\mu\nu}$ include the usual matter stress tensor and the contributions from the higher curvature terms. The invariance of the stress tensor $T_{\mu\nu}$ sets a non-trivial constraint on ϕ_i. The simplest constraint is to require that the fields themselves are invariant $\mathcal{L}_v \phi_i = 0$. However, this is not strictly necessary if we want $T_{\mu\nu}$ to be invariant. In [24], it was demanded that any gauge invariant observables must be invariant under the full symmetry. In the next subsection we will see that such a requirement can also be released.

Now let us focus on the Einstein equations. The left hand side is automatically a conserved two-tensor invariant under the Lifshitz symmetries. Thus it can be written in the form (2.4). It can be seen that α and β are simple functions of z and L, whose explicit expressions will not be shown. On the other hand, the stress tensor takes the following form

$$T_{\mu\nu} = \alpha(z, L, \phi_i) \frac{dt^2}{r^{2z}} + \beta(z, L, \phi_i) \frac{d\vec{x}^2}{r^2} + \left[\frac{(d-2)\beta(z, L, \phi_i) - z\alpha(z, L, \phi_i)}{d-2+z} \right] \frac{dr^2}{r^2}. \tag{2.7}$$

Then the Einstein equations reduce to

$$\alpha = \alpha(z, L, \phi_i), \qquad \beta = \beta(z, L, \phi_i). \tag{2.8}$$

Once the values of ϕ_i are fixed by the ϕ equations of motion, the above equations can be seen as two equations for L and z.

Thus we can conclude as follows: The Lifshitz symmetry of the spacetime will be deformed (by changing the value of z) but not broken once higher order corrections are incorporated. In particular, if we can find a Lifshitz spacetime with certain z and L for one action, then for any small deformations of the parameters in the action we may find another solution with z', L' which are nearby values of z and L. Conversely, variations of the action can only renormalize the parameters z and L in the metric.

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2.2 Solutions in asymptotically Lifshitz spacetimes

We will review the solutions which are asymptotic to Lifshitz metric obtained in [22], including both zero-temperature and black brane cases. Consider the following action in $(d+2)$-dimensional spacetime (without higher derivative corrections)

$$S = \frac{1}{16\pi G_{d+2}} \int d^{d+2}x\sqrt{-g}\Big[R - 2\Lambda - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{\lambda\phi}F_{\mu\nu}F^{\mu\nu}\Big], \tag{2.9}$$

where Λ is the cosmological constant and the matter fields are a massless scalar and an abelian gauge field.

Such a theory admits the following zero-temperature solution whose metric is Lifshitz-like

$$\begin{aligned} ds^2 &= L^2\Big(-r^{2z}dt^2 + \frac{dr^2}{r^2} + r^2\sum_{i=1}^{d} dx_i^2\Big), \\ F_{rt} &= qe^{-\lambda\phi}r^{z-d-1}, & e^{\lambda\phi} &= r^{\lambda\sqrt{2(z-1)d}}, \\ \lambda^2 &= \frac{2d}{z-1}, & q^2 &= 2L^2(z-1)(z+d), \\ \Lambda &= -\frac{(z+d-1)(z+d)}{2L^2} \end{aligned} \tag{2.10}$$

as well as black brane solution

$$ds^2 = L^2\Big(-r^{2z}f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2\sum_{i=1}^{d} dx_i^2\Big), \quad f(r) = 1 - \frac{r_+^{z+d}}{r^{z+d}}, \tag{2.11}$$

where the other fields in the black brane solution remain the same as those in the zero-temperature solution. It is found that the AdS spacetime is also a solution to the equations of motion with $\phi = 0$ and $F_{rt} = 0$. Although the above metrics are Lifshitz-like or asymptotically Lifshitz-like, such solutions cannot be thought of as genuine gravity duals of Lifshitz fixed points, as the dilaton is not constant. However, such solutions are of interest themselves due to the black brane solution. We can study the thermodynamic and hydrodynamic properties of such black branes, which may be of help in understanding the genuine gravity duals of Lifshitz fixed points at finite temperature.

In the last subsection, it was required that the gauge invariant observables must be invariant under the full Lifshitz symmetry. Here we can see that such a requirement can be released due to the coupling between the dilaton and the gauge fields. The stress tensor is

$$T_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}\Big(\frac{1}{2}\partial_\rho\phi\ \partial^\rho\phi + \frac{1}{4}e^{\lambda\phi}F_{\rho\sigma}F^{\rho\sigma}\Big) + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}e^{\lambda\phi}F_{\mu\rho}F_\nu{}^\rho. \tag{2.12}$$

The components of the stress tensor can be obtained by substituting the values of the matter fields in (2.10)

$$T_{tt} = (z-1)\Big(\frac{z}{2} + d\Big)r^{2z}, \quad T_{rr} = -\frac{z(z-1)}{2r^2}, \quad T_{ii} = \frac{1}{2}z(z-1)r^2, \tag{2.13}$$

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which are all invariant under the Lifshitz symmetries. Once higher derivative corrections are incorporated, the equations of motion of the matter fields do not change and the additional part of the stress tensor is comprised of the Riemann tensor of the background geometry. Thus the stress tensor is still invariant under the Lifshitz symmetries.

3 R^2 corrections to zero-temperature backgrounds

We shall study R^2 corrections to the zero-temperature Lifshitz geometry in this section. Firstly we will obtain a solution in pure Gauss-Bonnet gravity and then we will consider the action appearing in (2.9) plus Gauss-Bonnet corrections. It should be emphasized that both of the solutions are exact, while we will investigate R^2 corrected black brane solutions in the next section by perturbative methods.

The general action containing the curvature squared corrections can be written as

$$S = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\Big[R - 2\Lambda + L^2(\alpha_1 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^2)\Big] + S_m, \quad (3.1)$$

where α_i are arbitrary small coefficients and S_m denotes the matter sector of the action. One specific model-Gauss-Bonnet gravity-has provided many interesting results, whose action is

$$S = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left[R - 2\Lambda + \frac{\lambda_{\rm GB}}{2}L^2(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2)\right] + S_m. \quad (3.2)$$

Several exact solutions of black holes in Gauss-Bonnet gravity have been obtained, see e.g. [25, 26]. FRW-like solutions and black holes for general five-dimensional R^2 gravity were studied in [27]. The conformal anomaly from higher derivative gravity in AdS/CFT correspondence was studied in [28]. From now on we will focus on five-dimensional case as the Gauss-Bonnet corrections are topological in four-dimensional spacetime and do not play an important role. The Einstein equations derived from (3.2) are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda g_{\mu\nu} + T^{\rm M}_{\mu\nu} + T^{\rm R}_{\mu\nu}, \quad (3.3)$$

where $T^{\rm M}_{\mu\nu}$ stands for the stress tensor of the matter sector and $T^R_{\mu\nu}$ comes from the Gauss-Bonnet term

$$\begin{aligned} T^{\rm R}_{\mu\nu} = \frac{\lambda_{\rm GB}}{2}L^2\bigg[&\frac{1}{2}g_{\mu\nu}(R_{\gamma\delta\lambda\sigma}R^{\gamma\delta\lambda\sigma} - 4R_{\gamma\delta}R^{\gamma\delta} + R^2) - 2RR_{\mu\nu} \\ &+4R_{\mu\gamma}R^{\gamma}{}_{\nu} + 4R^{\gamma\delta}R_{\gamma\mu\delta\nu} - 2R_{\mu\gamma\delta\lambda}R_{\nu}{}^{\gamma\delta\lambda}\bigg]. \end{aligned} \quad (3.4)$$

3.1 Solutions in pure Gauss-Bonnet gravity

First let us consider Lifshitz-like solutions in pure Gauss-Bonnet gravity, i.e.without introducing matter fields. The Einstein equations (3.3) turn out to be

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda g_{\mu\nu} + T^{\rm R}_{\mu\nu}, \quad (3.5)$$

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where $T^{\rm R}_{\mu\nu}$ has been given in (3.4). The Lifshitz background can be written as

$$ds^2 = L^2\left[-\frac{dt^2}{r^{2z}} + \frac{1}{r^2}(dr^2 + dx_1^2 + dx_2^2 + dx_3^2)\right]. \tag{3.6}$$

The non-vanishing components of the Ricci tensor are

$$R_{tt} = \frac{z(z+3)}{r^{2z}}, \quad R_{rr} = -\frac{z^2+3}{r^2}, \quad R_{ii} = -\frac{z+3}{r^2}, \quad i = 1,2,3, \tag{3.7}$$

and the non-vanishing components of the stress tensor

$$T^{\rm R}_{tt} = -\frac{6\lambda_{\rm GB}}{r^2}, \quad T^{\rm R}_{rr} = \frac{6\lambda_{\rm GB}z}{r^2}, \quad T^{\rm R}_{ii} = \frac{2\lambda_{\rm GB}z(z+2)}{r^2}. \tag{3.8}$$

Thus we can easily find that the Lifshitz background is a solution to the Einstein equations when the Gauss-Bonnet coupling constant $\lambda_{\rm GB}$ and the cosmological constant Λ take the following values

$$\lambda_{\rm GB} = \frac{1}{2}, \quad \Lambda = -\frac{3}{L^2}. \tag{3.9}$$

Here are some remarks on this solution:

- When z=1, the metric reduces to AdS, which is a solution to the Einstein equations for any values of $\lambda_{\rm GB}$ and Λ.

- In the literatures studying Gauss-Bonnet black holes in AdS and dS spacetimes [26], in order to obtain a meaningful black hole solution, the Gauss-Bonnet coupling constant $\lambda_{\rm GB}$ should have an upper bound $\lambda_{\rm GB}^{\rm upper} = 1/4$. Here $\lambda_{\rm GB}$ goes beyond this bound. We will go back to this issue when discussing the ratio of shear viscosity to entropy density.

- In [24], an exact solution of Lifshitz geometry was obtained in pure R^2 gravity, with the coefficient in front of the R^2 term and the cosmological constant

  $$c_1 = \frac{L^2}{2z^2+4z+6}, \quad \Lambda = \frac{z^2+2z+3}{L^2}.$$

  We can see that when the space is large, so is the higher order corrections, and vice versa. This invalidates the perturbative description, as these solutions balance the curvature terms against curvature squared terms, the quadratic approximation cannot be expected to be reliable. Here we have similar situations and just as [24], we can also expect that a non-trivial matter sector may solve this problem.

- In [25, 26], exact solutions of black holes in pure Gauss-Bonnet gravity were obtained. However, due to the Birkhoff theorem, we cannot find exact solutions of black holes with anisotropic scaling symmetry in pure Gauss-Bonnet gravity.

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3.2 Solutions with non-trivial matter fields

We will consider Lifshitz-like solutions with non-trivial matter fields in this subsection. To be concrete, we shall add Gauss-Bonnet corrections to the effective action (2.9) and try to find the corresponding solutions. Note that here the solutions are exact while we will study the corrections in the black brane case by perturbative methods in the next section.

Now the Einstein equations are given by (3.3)

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda g_{\mu\nu} + T^{\mathrm{M}}_{\mu\nu} + T^{\mathrm{R}}_{\mu\nu},$$

where the stress tensor of the matter sector is

$$T^{\mathrm{M}}_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\,\partial^\rho\phi + \frac{1}{4}e^{\lambda\phi}F_{\rho\sigma}F^{\rho\sigma}\right) + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}e^{\lambda\phi}F_{\mu\rho}F_\nu{}^\rho$$

and $T^{\mathrm{R}}_{\mu\nu}$ has been shown in (3.4). In addition, the equations of motion for the matter fields are

$$\partial_\mu\left(\sqrt{-g}e^{\lambda\phi}F^{\mu\nu}\right) = 0, \tag{3.10}$$

$$\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) - \frac{\lambda}{4}\sqrt{-g}e^{\lambda\phi}F_{\mu\nu}F^{\mu\nu} = 0. \tag{3.11}$$

Here we still make the ansatz for the metric

$$ds^2 = L^2\left[-\frac{dt^2}{r^{2z}} + \frac{1}{r^2}(dr^2 + dx_1^2 + dx_2^2 + dx_3^2)\right].$$

After solving the equations of motion, we can find the following solution

$$\begin{aligned}
&\phi = \pm\sqrt{6(1-2\lambda_{\mathrm{GB}})(z-1)}\log r, && F_{rt} = qe^{-\lambda\phi}r^{2-z},\\
&\lambda^2 = \frac{6}{(1-2\lambda_{\mathrm{GB}})(z-1)}, && q^2 = 2(1-2\lambda_{\mathrm{GB}})(z-1)(z+3)L^2,\\
&\Lambda = -\frac{(z+2)(z+3)}{2L^2} + \frac{\lambda_{\mathrm{GB}}z(z+5)}{L^2}.
\end{aligned} \tag{3.12}$$

Note that here the dynamical exponent z has been renormalized. Furthermore, λ_{GB} should have an upper bound $1/2$ to ensure a physical solution. Let us denote $z = z_0 + \delta z$ where z_0 is the dynamical exponent in the Einstein-matter theory. By fixing the cosmological constant Λ and solving the linearized equations, we can arrive at

$$\delta z = \frac{2\lambda_{\mathrm{GB}}z_0(z_0+5)}{(1-2\lambda_{\mathrm{GB}})(2z_0+5)}. \tag{3.13}$$

The $z_0 = 1$ case should be treated separately. As can be seen from the unperturbed solution (2.10), when $z_0 = 1$, both the dilaton and the gauge field strength vanish, then the theory reduces to pure Einstein gravity. The non-renormalization of the AdS case is a well known fact. Furthermore, in [24], when considering four-dimensional Lifshitz spacetime, it was found that the $z_0 = 2$ case seemed to be protected. But there was no sign of an extra symmetry protecting this solution and it could be renormalized under certain ad-hoc higher order terms. Here we can see that z can also be renormalized in $z_0 = 2$ case, so $z_0 = 2$ is nothing special compared to other cases, which supports their argument.

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4 R^2 corrections to black branes

In this section we consider Gauss-Bonnet corrections to black branes. Unfortunately, it is quite difficult to find an exact solution which is asymptotic to Lifshitz spacetime in Gauss-Bonnet gravity. Then we have to solve the equations of motion perturbatively, following [29].

Considering the following action

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\Big[R - 2\Lambda - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{\lambda\phi}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda_{\rm GB}}{2}L^2(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2)\Big], \quad (4.1)$$

the corresponding equations of motion remain the same as before

$$\begin{aligned}
R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= -\Lambda g_{\mu\nu} + T^{\rm M}_{\mu\nu} + T^{\rm R}_{\mu\nu},\\
T^{\rm M}_{\mu\nu} &= -\frac{1}{2}g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\,\partial^\rho\phi + \frac{1}{4}e^{\lambda\phi}F_{\rho\sigma}F^{\rho\sigma}\right) + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}e^{\lambda\phi}F_{\mu\rho}F_\nu{}^\rho,\\
T^{\rm R}_{\mu\nu} &= \frac{\lambda_{\rm GB}}{2}L^2\Big[\frac{1}{2}g_{\mu\nu}(R_{\gamma\delta\lambda\sigma}R^{\gamma\delta\lambda\sigma} - 4R_{\gamma\delta}R^{\gamma\delta} + R^2) - 2RR_{\mu\nu}\\
&\qquad + 4R_{\mu\gamma}R^\gamma{}_\nu + 4R^{\gamma\delta}R_{\gamma\mu\delta\nu} - 2R_{\mu\gamma\delta\lambda}R_\nu{}^{\gamma\delta\lambda}\Big],\\
\partial_\mu(\sqrt{-g}e^{\lambda\phi}F^{\mu\nu}) &= 0,\\
\partial_\mu(\sqrt{-g}\partial^\mu\phi) - \frac{\lambda}{4}\sqrt{-g}e^{\lambda\phi}F_{\mu\nu}F^{\mu\nu} &= 0.
\end{aligned} \quad (4.2)$$

Note that the equations of motion for the matter fields do not change after incorporating the Gauss-Bonnet corrections.

Let us focus on the right hand side of the Einstein equations. Since we are trying to solve the equations at the leading order of $\lambda_{\rm GB}$, we can substitute the unperturbed metric

$$ds^2 = L^2\Big[-\frac{f(r)}{r^{2z}}dt^2 + \frac{dr^2}{r^2 f(r)} + \frac{1}{r^2}(dx_1^2 + dx_2^2 + dx_3^2)\Big], \quad f(r) = 1 - \frac{r^{z+3}}{r_+^{z+3}}$$

into $T^R_{\mu\nu}$. Furthermore, we neglect the backreactions of the Gauss-Bonnet corrections to the matter fields and substitute the unperturbed values of those fields into $T^{\rm M}_{\mu\nu}$. The ansatz for the metric is

$$ds^2 = \frac{L^2}{r^2}\Big[-e^{2a(r)}dt^2 + e^{-2b(r)}dr^2 + dx_1^2 + dx_2^2 + dx_3^2\Big]. \quad (4.3)$$

The components of the Ricci tensor can be combined as

$$R^t_t - R^r_r = \frac{3}{L^2}e^{2b(r)}r(a'(r) - b'(r)), \quad (4.4)$$

$$\frac{1}{3}(R^t_t - R^r_r) - R^x_x = -\frac{1}{L^2}\left(\frac{e^{2b(r)}}{r^4}\right)' r^5, \quad (4.5)$$

where the prime stands for derivative with respect to r. On the other hand, we have the following expressions due to the Einstein equations

$$R^t_t - R^r_r = T^{\mathrm{M}t}{}_t + T^{\mathrm{R}t}{}_t - T^{\mathrm{M}r}{}_r - T^{\mathrm{R}r}{}_r, \tag{4.6}$$

$$\frac{1}{3}(R^t_t - R^r_r) - R^x_x = \frac{2}{3}(T^{\mathrm{M}t}{}_t + T^{\mathrm{M}r}{}_r - \Lambda). \tag{4.7}$$

Therefore,

$$e^{2b(r)} = -\frac{2}{3}L^2 r^4 \left[\int \frac{dr}{r^5}(T^{\mathrm{M}t}{}_t + T^{\mathrm{M}r}{}_r - \Lambda) + \mathrm{const} \right], \tag{4.8}$$

$$a(r) = b(r) + \frac{L^2}{3}\int \frac{dr}{r} e^{-2b(r)}(T^{\mathrm{M}t}{}_t + T^{\mathrm{R}t}{}_t - T^{\mathrm{M}r}{}_r - T^{\mathrm{R}r}{}_r). \tag{4.9}$$

After substituting the unperturbed metric and matter fields, we can obtain the following perturbative black brane solution

$$ds^2 = L^2\left[-\frac{f(r)}{r^{2z}}h(r)dt^2 + \frac{dr^2}{r^2 f(r)} + \frac{1}{r^2}(dx_1^2 + dx_2^2 + dx_3^2)\right], \tag{4.10}$$

where

$$z = z_0 + 2\lambda_{\mathrm{GB}}(z_0 - 1), \qquad h(r) = \exp\left[4\lambda_{\mathrm{GB}}\frac{z_0-1}{z_0+3}\left(\frac{r}{r_+}\right)^{z_0+3}\right],$$
$$f(r) = 1 - \left(\frac{r}{r_+}\right)^{z_0+3} + \lambda_{\mathrm{GB}}\left[1 - \left(\frac{r}{r_+}\right)^{z_0+3}\right]^2. \tag{4.11}$$

One can check that this solution agrees with the one appearing in [29] when $z_0 = 1$. For general cases, the horizon still locates at $r = r_+$ and the Hawking temperature is

$$T_H = \frac{1}{4\pi}\frac{z_0+3}{r_+^z}\left(1 + 2\lambda_{\mathrm{GB}}\frac{z_0-1}{z_0+3}\right). \tag{4.12}$$

For black branes in Gauss-Bonnet gravity, the area law for entropy still holds [30], then

$$S_{\mathrm{BH}} = \frac{1}{4\pi}\frac{L^3 V_3}{r_+^3}, \tag{4.13}$$

where V_3 denotes the volume of the spatial directions.

5 Calculating η/s

The AdS/CFT correspondence has provided us an efficient way to study the dynamics of strongly coupled gauge theories. One remarkable example is the calculation of the ratio of the shear viscosity over entropy density η/s. It has been found that

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

is a universal result for all gauge theories with Einstein gravity duals in the large N limit. Furthermore, it was conjectured that $1/4\pi$ is a universal lower bound for all materials, which is known as the KSS bound [31]. Later the authors of [29, 32, 33] calculated the ratio in R^2 gravity and found that the lower bound was violated. A new lower bound $4/25\pi$ was proposed in [33] by considering the causality of the dual field theory. For more discussions on violation of the KSS bound in higher derivative gravity, see [34].

It was conjectured that the shear viscosity is completely determined by the effective coupling of the transverse gravitons on the horizon in the dual gravity description [35]. This was confirmed in [36] via the scalar membrane paradigm and in [37] by calculating the on-shell action of the transverse gravitons. Such an effective action in a given background was assumed to be a minimally coupled massless scalar with an effective coupling which depends on the radial coordinate, while in Einstein gravity the effective coupling is a constant. However, this formalism is not covariant under coordinate transformations, then the coordinate system of the background geometry also affects the form of the action of transverse gravitons. In [38], a new formalism was proposed, where a new three-dimensional effective metric $\tilde{g}_{\mu\nu}$ was introduced and the transverse gravitons were minimally coupled to this new effective metric. The action in this new formalism can take a covariant form. Similar discussions on this issue were also presented in [39].

We shall calculate the shear viscosity of field theory with R^2 corrected black brane dual using the approach proposed in [38]. As a non-relativistic theory, the causality of the boundary field theory cannot be treated as a constraint on the lower bound of η/s.

5.1 Shear viscosity from the effective coupling of transverse gravitons

The shear viscosity can be calculated via the Kubo formula

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega i}\Big(G^A_{x_1x_2,x_1x_2}(\omega,0) - G^R_{x_1x_2,x_1x_2}(\omega,0)\Big), \tag{5.1}$$

where the retarded Green's function $G^R_{\mu\nu,\lambda\rho}$ is defined by

$$G^R_{\mu\nu,\lambda\rho} = -i\int d^4x e^{-ik\cdot x}\theta(t) < [T_{\mu\nu}(x), T_{\lambda\rho}(0)] >, \tag{5.2}$$

and the advanced Green's function satisfies $G^A_{\mu\nu,\lambda\rho}(k) = G^R_{\mu\nu,\lambda\rho}(k)^*$. These Green's functions are defined on the field theory side. According to the field-operator correspondence, such Green's functions can be calculated through the effective action of the gravitons of the dual gravity theory.

We can choose spatial coordinates so that the momentum of the perturbation points along the $x_3 \equiv z$ axis. Considering tensor perturbation $h_{12} = h_{12}(t,u,z)$ with u being the radial coordinate, we denote $\phi = h^1_2$ and write ϕ as $\phi(t,u,z) = \phi(u)e^{-i\omega t+ipz}$. For gravity theories in which the transverse gravitons can be decoupled from other perturbations, the effective bulk action of the transverse gravitons can be written in a general form

$$S = \frac{V_{1,2}}{16\pi G}\Big(-\frac{1}{2}\Big)\int d^3x\sqrt{-\tilde{g}}\Big(\tilde{K}(u)\tilde{g}^{MN}\tilde{\nabla}_M\phi\tilde{\nabla}_N\phi + m^2\phi^2\Big) \tag{5.3}$$

up to some total derivatives. Here $\tilde{g}_{MN}$ is a three-dimensional effective metric, m is an effective mass and $\tilde{\nabla}_M$ is the covariant derivative using $\tilde{g}_{MN}$. Notice that ϕ is a scalar in the three dimensions t, u, z, while it is not a scalar in the whole five dimensions. We write the action in the three-dimensional form so that it is general covariant and $\tilde{K}(u)$ is a scalar under general coordinate transformations. It should be pointed out that this is not the ordinary dimensional reduction. In the following we will use $g_{\mu\nu}$ to denote the whole five-dimensional background.

Recalling the corrected black brane metric in (4.10) and performing the following coordinate transformations

$$\rho = \frac{1}{r}, \quad \rho_+ = \frac{1}{r_+}, \quad \left(\frac{\rho_+}{\rho}\right)^{z_0+3} = u^2, \tag{5.4}$$

the black brane metric metric turns out to be

$$ds^2 = L^2\left[-g(u)(1-u)dt^2 + \frac{1}{h(u)(1-u)}du^2 + \frac{\rho_+^2}{u^A}(dx_1^2 + dx_2^2 + dx_3^2)\right], \tag{5.5}$$

where

$$\begin{aligned} g(u) &= \rho_+^{2z} u^{-\frac{4z}{z_0+3}}(1+u)(1+\lambda_{\rm GB}(1-u^2)) \exp\left[4\lambda_{\rm GB}\frac{z_0-1}{z_0+3}u^2\right], \\ h(u) &= \frac{1}{4}(z_0+3)^2 u^2(1+u)(1+\lambda_{\rm GB}(1-u^2)), \qquad\qquad A = \frac{4}{z_0+3}. \end{aligned} \tag{5.6}$$

Then the horizon of the black brane locates at $u = 1$ and the boundary locates at $u = 0$.

Following [38], we write down the action of the transverse gravitons in momentum space

$$S = \frac{V_{1,2}}{16\pi G}\left(-\frac{1}{2}\right)\int\frac{dwdp}{(2\pi)^2}du\sqrt{-\tilde{g}}\Big[\tilde{K}(u)(\tilde{g}^{uu}\phi'\phi' + w^2\tilde{g}^{tt}\phi^2 + p^2\tilde{g}^{zz}\phi^2) + m^2\phi^2\Big], \tag{5.7}$$

where

$$\begin{aligned} \phi(t,u,z) &= \int\frac{dwdp}{(2\pi)^2}\phi(u;k)e^{-iwt+ipz}, \\ k &= (w,0,0,p), \qquad\qquad \phi(u;-k) = \phi^*(u;k), \end{aligned} \tag{5.8}$$

and the prime denotes derivative with respect to u. The corresponding equation of motion is given by

$$\phi''(u;k) + A(u)\phi'(u;k) + B(u)\phi(u;k) = 0, \tag{5.9}$$

where

$$A(u) = \frac{(\sqrt{-\tilde{g}}\tilde{K}(u)\tilde{g}^{uu})'}{\sqrt{-\tilde{g}}\tilde{K}(u)\tilde{g}^{uu}}, \quad B(u) = -\tilde{g}_{uu}\left(\tilde{g}^{tt}w^2 + \tilde{g}^{zz}p^2 + \frac{m^2}{\tilde{K}(u)}\right). \tag{5.10}$$

Furthermore, by repeating the calculations in section II of [38], we can find that here we still have the following formula for η

$$\eta = \frac{1}{16\pi G}(\sqrt{\tilde{g}_{zz}}\tilde{K}(u))|_{u=1}. \tag{5.11}$$

Next we shall calculate the effective action of the transverse gravitons on the background (5.5). From the first order Einstein equations we can see that the transverse gravitons can get decoupled from other perturbations. Then we can obtain the effective action of the transverse gravitons by keeping quadratic terms of ϕ in the original action (4.1). The action can be written in the form of (5.7) with three-dimensional effective metric

$$\tilde{g}^{uu} = \left(1 + \frac{\lambda_{\rm GB}}{2}\frac{Ag'_{tt}g^{uu}}{ug_{tt}}\right)g^{uu}, \tag{5.12}$$

$$\tilde{g}^{tt} = \left[1 + \frac{\lambda_{\rm GB}}{2}\left(\frac{Ag'^{uu}}{u} - \frac{(A^2+2A)g^{uu}}{u^2}\right)\right]g^{tt}, \tag{5.13}$$

$$\tilde{g}^{zz} = \left[1 + \frac{\lambda_{\rm GB}}{2}\left(\frac{g'^2_{tt}g^{uu}}{g^2_{tt}} - \frac{g'_{tt}g'^{uu}}{g_{tt}} - \frac{2g^{uu}g''_{tt}}{g_{tt}}\right)\right]g^{zz}. \tag{5.14}$$

The m^2 term vanishes due to the Einstein equations of the background metric. Note that when $z_0 = 1$, i.e. $A = 1$, the above expressions agree with those obtained in [38] with vanishing dilaton field.

By using the formula $\tilde{K}(u) = \sqrt{-g}/\sqrt{-\tilde{g}}$ and recalling the fact that the area law still holds for black branes in Gauss-Bonnet gravity, we can arrive at the final result

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{\lambda_{\rm GB}}{2}Ah(1)\right] = \frac{1}{4\pi}[1-(z_0+3)\lambda_{\rm GB}]. \tag{5.15}$$

Here are some remarks on this result:

- When $z_0 = 1$, the black brane metric is asymptotically AdS. We can recover the result obtained in [32].
- For general $z_0 \neq 1$, in order to obtain a non-vanishing η/s, $\lambda_{\rm GB}$ should have an upper bound $1/(z_0+3)$. The upper bound of $\lambda_{\rm GB}$ was discussed in [40] where it was found to be 1/4 by the constraints of causality and stability. Here a similar upper bound in non-relativistic theory requires further understanding.
- In the literatures discussing the ratio of shear viscosity over entropy density in higher derivative theory of gravity, the new lower bound of η/s–$4/25\pi$-can be obtained by considering the causality of the boundary field theory. However, here we cannot take such a constraint as the dual field theory is non-relativistic. We will address it in detail in next subsection.

5.2 Causality cannot be a constraint

It was discovered that the KSS bound can be violated in R^2 gravity, but the causality of the boundary field theory can constrain the parameters and introduce a new lower bound [32, 33]. But here we will see that causality cannot be a constraint in a non-relativistic theory.

Following [38], we can transform the action of the transverse gravitons into a minimally coupled form

$$S = \frac{V_{1,2}}{16\pi G}-\frac{1}{2}\int d^3x\sqrt{-\bar{g}}\left(\bar{g}^{MN}\partial_M\phi\partial_N\phi + \bar{m}^2\phi^2\right), \tag{5.16}$$

with

$$\bar{g}^{MN} = \tilde{K}(u)^{-2}\tilde{g}^{MN}, \quad \bar{m}^2 = \tilde{K}(u)^{-3}m^2. \tag{5.17}$$

For the case at hand, $\bar{m}^2$ term vanishes, so the equation of motion is

$$\bar{g}^{MN}\bar{\nabla}_M\bar{\nabla}_N\phi = 0. \tag{5.18}$$

Then we can apply the geometrical optics approximation in the large momentum limit. The wave function is written in the form $\phi = \phi_{\text{en}}(t,u,z)e^{i\theta(t,u,z)}$, where ϕ_{en} stands for a slowly changing envelope function and θ is a rapidly varying phase function. Expanding the equation of motion at leading order, we obtain

$$\frac{dx^M}{ds}\frac{dx^N}{ds}\bar{g}_{MN} = 0, \tag{5.19}$$

where $dx^M/ds = \bar{g}^{MN}\bar{\nabla}_N\theta$.

Due to translation symmetries in t and z directions, $\omega = i\bar{\nabla}_t\theta$ and $q = -i\bar{\nabla}_z\theta$ are still conserved integrals of motion along the geodesic. Then (5.19) can be written as

$$\left(\frac{du}{ds}\right)^2 = (-\bar{g}^{tt}\bar{g}^{uu}q^2)\left[\frac{\omega^2}{q^2} - \frac{\bar{g}^{zz}}{-\bar{g}^{tt}}\right]. \tag{5.20}$$

If we assume $q^2 > 0$ and denote $\tilde{s} = s\sqrt{-\bar{g}^{tt}\bar{g}^{uu}q^2}$, we can get

$$\left(\frac{du}{d\tilde{s}}\right)^2 = \frac{\omega^2}{q^2} - \frac{\bar{g}^{zz}}{-\bar{g}^{tt}}. \tag{5.21}$$

This equation describes a one-dimensional system with a particle of energy $\frac{\omega^2}{q^2}$ moving in a potential $\frac{\bar{g}^{zz}}{-\bar{g}^{tt}}$. The effective geometry can be expressed as

$$ds^2 = \bar{g}_{MN}dx^M dx^N = -\bar{g}_{tt}\left(-dt^2 + \frac{1}{c_g^2}dz^2\right) + \bar{g}_{uu}du^2, \tag{5.22}$$

where

$$c_g^2 = \frac{\bar{g}^{zz}}{-\bar{g}^{tt}} = \frac{\tilde{g}^{zz}}{-\tilde{g}^{tt}} \tag{5.23}$$

denotes the local "speed of graviton" on constant u hypersurface.

In the relativistic cases, $c_g^2 = 0$ on the horizon and $c_g^2 = 1$ at infinity. One can expand c_g^2 near the boundary and require that it should be smaller than one to avoid causality violation. This requirement gives a new lower bound on η/s. But here the situation is quite different, as it can be easily seen that $c_g^2 \to \infty$ at the boundary $u = 0$ by using (5.13) and (5.14). This should not be surprising as the boundary field theory is non-relativistic. So we cannot take causality as a constraint on η/s.

6 Summary and discussion

We study R^2 corrections to asymptotically Lifshitz spacetimes in five dimensions. For the zero-temperature background, we obtain exact solutions both in pure Gauss-Bonnet gravity and in Gauss-Bonnet gravity with non-trivial matter. In the latter case we find that the dynamical exponent z undergoes a finite renormalization. For the finite-temperature

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background, we obtain perturbative solutions in Gauss-Bonnet theory with non-trivial matter. The ratio of shear viscosity over entropy density is also calculated. It violates the KSS bound but here causality cannot be treated as a constraint due to the non-relativistic nature of the boundary theory.

The violation of the KSS bound in non-relativistic theory with higher derivative correction was already observed in [24], where they obtained

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right) \tag{6.1}$$

for asymptotically Schrödinger black holes. Here N denotes the rank of the gauge group and this result is the same as the relativistic counterparts. Since the near horizon geometry of the Schrödinger black holes is the same as that of usual AdS black holes, one can safely obtain the above result following the arguments in [36]. However, the result obtained in this paper is different from the relativistic case. According to [36], this is natural as the Lifshitz black branes and the AdS counterparts have different near horizon structures. Furthermore, due to the difficulty of embedding the Lifshitz background into string/M theory [17], the corrections to η/s for Lifshitz black branes are difficult to evaluate in the context of string /M theory.

Recently there have been several interesting discussions on higher derivative corrections to η/s, see e.g. [41–44]. The causality of the boundary field theory plays an important role in constraining η/s. Here causality cannot be constraint since the boundary field theory is non-relativistic. However, as argued in [24], the observations in [45] would suggest that the problem with violations of KSS bound are as much about unitarity and locality as about causality, and should persist in the non-relativistic limit. The unitarity and locality might be served as new constraints on the KSS bound in non-relativistic theory and it would be interesting to investigate this topic in future.

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References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200] [SPIRES].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [SPIRES].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [SPIRES].

[4] D.T. Son and A.O. Starinets, *Viscosity, black holes and quantum field theory*, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 95 [arXiv:0704.0240] [SPIRES].

[5] S.A. Hartnoll, *Lectures on holographic methods for condensed matter physics*, arXiv:0903.3246 [SPIRES];
C.P. Herzog, *Lectures on holographic superfluidity and superconductivity*, *J. Phys.* **A 42** (2009) 343001 [arXiv:0904.1975] [SPIRES];
J. McGreevy, *Holographic duality with a view toward many-body physics*, arXiv:0909.0518 [SPIRES].

[6] D.T. Son, *Toward an AdS/cold atoms correspondence: a geometric realization of the Schrödinger symmetry*, *Phys. Rev.* **D 78** (2008) 046003 [arXiv:0804.3972] [SPIRES].

[7] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, *Phys. Rev. Lett.* **101** (2008) 061601 [arXiv:0804.4053] [SPIRES].

[8] C.P. Herzog, M. Rangamani and S.F. Ross, *Heating up galilean holography*, *JHEP* **11** (2008) 080 [arXiv:0807.1099] [SPIRES].

[9] J. Maldacena, D. Martelli and Y. Tachikawa, *Comments on string theory backgrounds with non-relativistic conformal symmetry*, *JHEP* **10** (2008) 072 [arXiv:0807.1100] [SPIRES].

[10] A. Adams, K. Balasubramanian and J. McGreevy, *Hot spacetimes for cold atoms*, *JHEP* **11** (2008) 059 [arXiv:0807.1111] [SPIRES].

[11] S.A. Hartnoll and K. Yoshida, *Families of IIB duals for nonrelativistic CFTs*, *JHEP* **12** (2008) 071 [arXiv:0810.0298] [SPIRES].

[12] E.G. Gimon, A. Hashimoto, V.E. Hubeny, O. Lunin and M. Rangamani, *Black strings in asymptotically plane wave geometries*, *JHEP* **08** (2003) 035 [hep-th/0306131] [SPIRES].

[13] S. Kachru, X. Liu and M. Mulligan, *Gravity duals of Lifshitz-like fixed points*, *Phys. Rev.* **D 78** (2008) 106005 [arXiv:0808.1725] [SPIRES].

[14] S. Sekhar Pal, *Towards gravity solutions of AdS/CMT*, arXiv:0808.3232 [SPIRES]; *More gravity solutions of AdS/CMT*, arXiv:0809.1756 [SPIRES]; *Anisotropic gravity solutions in AdS/CMT*, arXiv:0901.0599 [SPIRES]; *Non-relativistic supersymmetric Dp-branes*, arXiv:0904.3620 [SPIRES].

[15] T. Azeyanagi, W. Li and T. Takayanagi, *On string theory duals of Lifshitz-like fixed points*, *JHEP* **06** (2009) 084 [arXiv:0905.0688] [SPIRES].

[16] M. Fujita, W. Li, S. Ryu and T. Takayanagi, *Fractional quantum Hall effect via holography: chern-simons, edge states and hierarchy*, *JHEP* **06** (2009) 066 [arXiv:0901.0924] [SPIRES].

[17] W. Li, T. Nishioka and T. Takayanagi, *Some no-go theorems for string duals of non-relativistic Lifshitz-like theories*, arXiv:0908.0363 [SPIRES].

[18] U.H. Danielsson and L. Thorlacius, *Black holes in asymptotically Lifshitz spacetime*, *JHEP* **03** (2009) 070 [arXiv:0812.5088] [SPIRES].

[19] R.B. Mann, *Lifshitz topological black holes*, *JHEP* **06** (2009) 075 [arXiv:0905.1136] [SPIRES].

[20] G. Bertoldi, B.A. Burrington and A. Peet, *Black holes in asymptotically Lifshitz spacetimes with arbitrary critical exponent*, arXiv:0905.3183 [SPIRES].

[21] G. Bertoldi, B.A. Burrington and A.W. Peet, *Thermodynamics of black branes in asymptotically Lifshitz spacetimes*, arXiv:0907.4755 [SPIRES].

JHEP10(2009)031

[22] M. Taylor, *Non-relativistic holography*, arXiv:0812.0530 [SPIRES].

[23] D.-W. Pang, *A note on black holes in asymptotically Lifshitz spacetime*, arXiv:0905.2678 [SPIRES].

[24] A. Adams, A. Maloney, A. Sinha and S.E. Vazquez, $1/N$ *effects in non-relativistic gauge-gravity duality*, *JHEP* **03** (2009) 097 [arXiv:0812.0166] [SPIRES].

[25] S. Nojiri and S.D. Odintsov, *Anti-de Sitter black hole thermodynamics in higher derivative gravity and new confining-deconfining phases in dual CFT*, *Phys. Lett.* **B 521** (2001) 87 [*Erratum ibid.* **B 542** (2002) 301] [hep-th/0109122] [SPIRES];
M. Cvetič, S. Nojiri and S.D. Odintsov, *Black hole thermodynamics and negative entropy in deSitter and anti-deSitter Einstein-Gauss-Bonnet gravity*, *Nucl. Phys.* **B 628** (2002) 295 [hep-th/0112045] [SPIRES];
S. Nojiri, S.D. Odintsov and S. Ogushi, *Friedmann-Robertson-Walker brane cosmological equations from the five-dimensional bulk (A)dS black hole*, *Int. J. Mod. Phys.* **A 17** (2002) 4809 [hep-th/0205187] [SPIRES].

[26] R.-G. Cai, *Gauss-Bonnet black holes in AdS spaces*, *Phys. Rev.* **D 65** (2002) 084014 [hep-th/0109133] [SPIRES];
R.-G. Cai and Q. Guo, *Gauss-Bonnet black holes in dS spaces*, *Phys. Rev.* **D 69** (2004) 104025 [hep-th/0311020] [SPIRES].

[27] S. Nojiri and S.D. Odintsov, *Brane-world cosmology in higher derivative gravity or warped compactification in the next-to-leading order of AdS/CFT correspondence*, *JHEP* **07** (2000) 049 [hep-th/0006232] [SPIRES];
S. Nojiri, S.D. Odintsov and S. Ogushi, *Cosmological and black hole brane world universes in higher derivative gravity*, *Phys. Rev.* **D 65** (2002) 023521 [hep-th/0108172] [SPIRES].

[28] S. Nojiri and S.D. Odintsov, *On the conformal anomaly from higher derivative gravity in AdS/CFT correspondence*, *Int. J. Mod. Phys.* **A 15** (2000) 413 [hep-th/9903033] [SPIRES].

[29] Y. Kats and P. Petrov, *Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory*, *JHEP* **01** (2009) 044 [arXiv:0712.0743] [SPIRES].

[30] T. Jacobson and R.C. Myers, *Black hole entropy and higher curvature interactions*, *Phys. Rev. Lett.* **70** (1993) 3684 [hep-th/9305016] [SPIRES].

[31] P. Kovtun, D.T. Son and A.O. Starinets, *Viscosity in strongly interacting quantum field theories from black hole physics*, *Phys. Rev. Lett.* **94** (2005) 111601 [hep-th/0405231] [SPIRES].

[32] M. Brigante, H. Liu, R.C. Myers, S. Shenker and S. Yaida, *Viscosity bound violation in higher derivative gravity*, *Phys. Rev.* **D 77** (2008) 126006 [arXiv:0712.0805] [SPIRES].

[33] M. Brigante, H. Liu, R.C. Myers, S. Shenker and S. Yaida, *The viscosity bound and causality violation*, *Phys. Rev. Lett.* **100** (2008) 191601 [arXiv:0802.3318] [SPIRES].

[34] I.P. Neupane and N. Dadhich, *Entropy bound and causality violation in higher curvature gravity*, *Class. Quant. Grav.* **26** (2009) 015013 [SPIRES];
X.-H. Ge, Y. Matsuo, F.-W. Shu, S.-J. Sin and T. Tsukioka, *Viscosity Bound, Causality Violation and Instability with Stringy Correction and Charge*, *JHEP* **10** (2008) 009 [arXiv:0808.2354] [SPIRES];
N. Banerjee and S. Dutta, *Shear Viscosity to Entropy Density Ratio in Six Derivative Gravity*, *JHEP* **07** (2009) 024 [arXiv:0903.3925] [SPIRES];
I.P. Neupane, *Black holes, entropy bound and causality violation*, *Int. J. Mod. Phys.* **A 24** (2009) 3584 [arXiv:0904.4805] [SPIRES].

JHEP10(2009)031

[35] R. Brustein, D. Gorbonos and M. Hadad, *Wald's entropy is equal to a quarter of the horizon area in units of the effective gravitational coupling*, *Phys. Rev.* **D 79** (2009) 044025 [arXiv:0712.3206] [SPIRES];
R. Brustein and A.J.M. Medved, *The ratio of shear viscosity to entropy density in generalized theories of gravity*, *Phys. Rev.* **D 79** (2009) 021901 [arXiv:0808.3498] [SPIRES];
The shear diffusion coefficient for generalized theories of gravity, *Phys. Lett.* **B 671** (2009) 119 [arXiv:0810.2193] [SPIRES].

[36] N. Iqbal and H. Liu, *Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm*, *Phys. Rev.* **D 79** (2009) 025023 [arXiv:0809.3808] [SPIRES].

[37] R.-G. Cai, Z.-Y. Nie and Y.-W. Sun, *Shear viscosity from effective couplings of gravitons*, *Phys. Rev.* **D 78** (2008) 126007 [arXiv:0811.1665] [SPIRES].

[38] R.-G. Cai, Z.-Y. Nie, N. Ohta and Y.-W. Sun, *Shear viscosity from gauss-bonnet gravity with a dilaton coupling*, *Phys. Rev.* **D 79** (2009) 066004 [arXiv:0901.1421] [SPIRES].

[39] N. Banerjee and S. Dutta, *Higher derivative corrections to shear viscosity from graviton's effective coupling*, *JHEP* **03** (2009) 116 [arXiv:0901.3848] [SPIRES].

[40] X.-H. Ge and S.-J. Sin, *Shear viscosity, instability and the upper bound of the Gauss-Bonnet coupling constant*, *JHEP* **05** (2009) 051 [arXiv:0903.2527] [SPIRES];
X.-H. Ge, S.-J. Sin, S.-F. Wu and G.-H. Yang, *Shear viscosity and instability from third order Lovelock gravity*, arXiv:0905.2675 [SPIRES].

[41] A. Buchel, R.C. Myers and A. Sinha, *Beyond $\eta/s = 1/4\pi$*, *JHEP* **03** (2009) 084 [arXiv:0812.2521] [SPIRES].

[42] S. Cremonini, K. Hanaki, J.T. Liu and P. Szepietowski, *Higher derivative effects on η/s at finite chemical potential*, *Phys. Rev.* **D 80** (2009) 025002 [arXiv:0903.3244] [SPIRES].

[43] A. Buchel and R.C. Myers, *Causality of holographic hydrodynamics*, arXiv:0906.2922 [SPIRES].

[44] D.M. Hofman, *Higher derivative gravity, causality and positivity of energy in a UV complete QFT*, *Nucl. Phys.* **B 823** (2009) 174 [arXiv:0907.1625] [SPIRES].

[45] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, *Causality, analyticity and an IR obstruction to UV completion*, *JHEP* **10** (2006) 014 [hep-th/0602178] [SPIRES].

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